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The Monetary Bonus System in Supplier Selections as Discriminated Borda Voting: Truth Telling by Vickrey-Clarke-Grove[1]

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Abstract: In business practice, monetary Bonus Systems are increasingly being used to base supplier decisions not only on price but also on other decision-relevant criteria. Monetary Bonus Systems differ from conventional score matrices in the natural weighting of the criteria among each other. However, the problem of truthful evaluation still arises for the individual criterion if this cannot be calculated hard. In addition to information deficits, internal stakeholders can also engage in tactical evaluation behavior. In this paper, we interpret the mechanism of supplier decision-making based on monetary Bonus Systems as a Discriminated Borda Voting mechanism between internal stakeholders and propose internal payments from stakeholders to the CFO, which turn this into an incentive-compatible mechanism according to Vickrey-Clarke-Grove. Finally, participation constraints and budget balancing is discussed briefly.

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1. Introduction

Utility analysis has always been used for supplier decisions in order to decide between suppliers or their offers not only on the basis of price, but taking into account all decision-relevant characteristics. Currently scoring systems are very common, where each relevant criterion is assigned a weight and each supplier is assigned a score in each criterion. If price is also treated as a criterion, finally the weighted sum of scores can be used as a decision criterion. The main criticism for this procedure is the extreme distortion of the relevance of the individual decision criteria. It is almost impossible to find a natural weighting.

This paper proposes the following solution for the problem: The projection of all decision-relevant criteria onto the monetary dimension in terms of Boni and Mali – the monetary “Bonus System”. Price is not treated as a criterion like all the others, but price remains the only true criterion for decision-making, and all other criteria are measured in their influence on the monetary dimension. This is referred to as the “Direct Price”, which is actually received by the supplier, and the “Comparable Price”, which is obtained after taking into account all Boni and Mali. To derive the Comparable Price from the Direct Price, Boni are deducted and Mali are added. In this paper we limit ourselves to considering absolute Boni and Mali. In general, there are also relative Boni and Mali, which act as percentages or factors between the Direct and Comparable Prices. Since we assume the Direct Prices to be static in this paper, relative Boni and Mali can be converted into absolute Boni and Mali without loss of generality.

In the Comparable Price, the customer is maximally price-sensitive. The supplier who prevails in the competition with the lowest Comparable Price will be paid the Direct Price resulting from the deduction of his Mali and the addition of his Boni.

As an example, we can consider evaluation matrices commonly used under the keyword “most economical offer” in the context of public procurement law. After decades of work with score matrices, these evaluation matrices are now also accepted in public procurement law as monetary Bonus Systems.

The elegant thing about the monetary Bonus System method is that you get a “natural” weighting between the criteria.

The problem, however, is that one might still not find the truth for the monetary evaluation of the individual criteria. There is no sharp demarcation between hard criteria, in which costs can be calculated, and soft criteria, in which, in extreme cases, purely personal preferences of the internal stakeholders (agents) play a role.

As a solution, analogous to the internal budget, which is awarded to a department if it can credibly prove that it will incur higher costs with a supplier – this practice already exists today: The supplier gets the corresponding Malus and if it prevails, it is not the supplier who gets paid the amount in question, but the department in question – conversely, one can also concede to an internal stakeholder (agent) that she will pay her justified preference for a supplier with an internal budget. Then she "buys" the Bonus for her preferred supplier. Only if this supplier wins, the stakeholder pays the Bonus from her budget, which the supplier ultimately receives explicitly. For the stakeholder, it has to pay off, otherwise her preference was not appropriate. In a way, this is just a new interpretation of the long-established practice of "soft" Boni that can be given by internal stakeholders.

The principle of "bought" soft Boni is ultimately a negotiation mechanism for internal decision-making. What is proposed here for supplier decisions (or has long been common practice) can also be transferred to other decision-making questions. When making decisions in the company, costs and benefits always play a role in the end and the principle can be applied in exactly the same way, even if there is no dynamic price negotiation with suppliers in the context of the decision in question.

In this paper we discuss several possibilities to compose such internal decisionmaking mechanism with internal payments and reach a conclusion that the preferred mechanism is a classical Vickrey-Clarke-Groves implementation which reveals truth telling of stakeholder's preferences.

In the next section, section two, we present the basic model. In section three we describe the current practice to address the supplier choice, which can be considered as a generalization of the Borda voting rule. In section four we introduce monetary payments. In section five we demonstrate how we can use Vickrey-Clarke-Groves mechanism to reach an optimal decision under incomplete information. Section six concludes.

2. The Model

There is a set X of potential suppliers of a demand. The set of suppliers represent the possible outcomes of our mechanism, the final outcome would be the supplier who will get the contract and will supply the demand in question. We assume that the price q_x of each supplier x is already known.

There are n agents – internal stakeholders, each of which has a set of outcome valuations. The valuation of stakeholder i is represented as a function $v_i: X \longrightarrow R$, which expresses the value it has for each supplier, in monetary terms. If $v_i(x) > 0$, we call this value a Bonus and if $v_i(x) < 0$, we call it a Malus for x according to the evaluation of stakeholder i.

Our goal is to select an outcome that maximizes the sum of stakeholders' values while considering the price of the suppler. In order to do that, we introduce one stakeholder, called CFO, with the following evaluation function:

$$v_{CFO}(x) = q_0 - q_x \qquad (1)$$

where q_0 is any arbitrary value. As you can see, CFO always prefers the least expensive supplier.

Then we define our goal function as

$$x^{opt}(v) = \arg\max_{x \in X} \sum_{i=1}^{n} v_i(x) \qquad (2)$$

You can see that if all evaluations equal, the cheapest supplier would win the demand. If another supplier is to be picked up in optimum, the price difference with the lowest price should be lower than the sum of stakeholders' valuations other than CFO.

Note that in practice when a Bonus System is used for a supplier decision, the so called "Comparable Price" of a supplier x is $-\sum_{i=1}^{n} v_i(x)$ and $x^{opt}(v) = \arg\min_{x \in X} \text{ComparabePrice}(x)$.

For an easier understanding of examples, one can assume, without loss of generality, that all Boni are positive (i.e. no Mali occur). Otherwise, all of a stakeholder's Boni and Mali $v_i(x), x \in X$ can be made positive by adding an appropriate constant value c to all of them without changing the supplier decision based on the sums $\sum_{i=1}^{n} v_i(x)$ in formula (2) which just increase synchronously for all suppliers x by that value c. For $i = CFO$ this is the case if $q_0 \geqq \max_{y \in X}(q_y)$, the price of the most expensive supplier.

In practice, Bonus systems are known as lists of decision criteria, each of which leads to an evaluation (Bonus or Malus) for each supplier. The real internal stakeholders are asked for their preferences regarding all or some of the criteria that fit their responsibilities. There is no loss of generality if we identify these criteria in this paper with the "ownership" of a single stakeholder each.

In an ideal world of the public information all valuations $v_i(x)$ would be common knowledge and the optimal choice of best supplier is done by solving a straightforward maximisation problem. In the real world, however, the stakeholder's valuation of the different suppliers is often based on soft information and is not easily verifiable. As a result, the stakeholder bid an announcement $\tilde{v}_i(x)$ which might deviate from the real valuation $v_i(x)$ out of tactical or whatever reasons and decision maker choice depends not on the real valuation $v_i(x)$ but on the announcements $\tilde{v}_i(x)$.

3. Current Practice: "Discriminated Borda Voting"

Currently, the most common practice in the real world is to ask all stakeholders about their valuations of the suppliers and make a decision, based on announcements alone, according to formula (2). This may be regarded as a kind of "Discriminated Borda Voting".

Following the classical Borda voting rule, also referred to as "Borda Counting" and introduced by Jean-Charles de Borda in Borda (1784), the voters submit a linear order over m alternatives. It prescribes that, for each voter, $m-1$ points are to be given to the top alternative, $m-2$ points to the second-to-top alternative, and so forth, with 0 points given to the alternative ranked last. The alternative with the largest sum of points across all voters wins. The decision procedure broadly used for the evaluating of suppliers can be considered as generalisation of the Borda voting rule, where every voter is not just simply announcing a rank of the suppliers, but proposes a discriminated Bonus for each supplier[2]

The resulting utility of all stakeholders (players) is equal to their valuation of the finally chosen supplier:

$$u_i := v_i\big(x^{opt}(\tilde{v})\big) \qquad (3)$$

We can easily demonstrate that such mechanism is not incentive compatible. Consider the following simple example. There are two suppliers, A and B, and two stakeholders, CFO and production division. CFO cares only about the supplier price and privately knows $q_A = 100$ and

[2] The authors would like to thank Manfred Holler for the idea of calling this solution "Discriminated Borda Voting."

$q_B = 110$. The production division cares about some other specific supplier's characteristics and evaluates possible savings from working with supplier B as 5. The resulting table represents values v for both stakeholders:

Supplier	A	B
CFO	10	0
Production	0	5

It is clear that truthful announcement is not an equilibrium: Assuming that CFO makes an announcement $\tilde{\boldsymbol{v}}_{\boldsymbol{CFO}}(\mathrm{A}) = \boldsymbol{v}_{\boldsymbol{CFO}}(\mathrm{A}) = \mathbf{10}$, the Production division has an incentive to deviate and announce $\tilde{\boldsymbol{v}}_{\boldsymbol{Prod}}(\mathrm{B}) > \mathbf{10}$ even if $\boldsymbol{v}_{\boldsymbol{Prod}}(\mathrm{B}) = \mathbf{5}$. In practice, the only reason not to announce arbitrarily high Boni is internal reporting and revision at place. This result also applies more generally to other scoring rules that are not incentive compatible (Baumeister et al. 2017).

4. Introducing payments

As we already mentioned earlier, an idea is to introduce possible payments for the stakeholders: making the announcement of a Bonus for a supplier they should be aware that as a result of the decision mechanism some payments would follow. It is assumed that the stakeholders have quasilinear utility functions; this means that, if the outcome is supplier x^{opt} and in addition the stakeholder receives a payment t_i, (positive or negative), then the total utility of stakeholder i is:

$$u_i \coloneqq v_i(x^{opt}) + t_i \qquad (4)$$

How should such payments be composed? The first idea which comes to the mind and that is occasionally already used in business practice is to define $t_i = -\tilde{v}_i(x^{opt})$. Each stakeholder should be ready to pay the monetary payment equal to his announcement for the finally selected supplier.

The problem with this solution is that it is still not incentive compatible. Even if it prevents from arbitrarily high Bonus announcements, it compares to a "pay-as-bid" auction as mechanism among the voting stakeholders i: Announcing $\tilde{v}_i(x) = v_i(x)$ for all suppliers x means to be sure that the final utility will become $u_i(x^{opt}) = 0$ regardless which supplier x becomes x^{opt}.

5. Vickrey-Clarke-Groves

In this section, we introduce a Vickrey-Clarke-Groves mechanism, which is both allocatively efficient and dominant strategy incentive compatible. The VCG mechanisms are named after their famous inventors William Vickrey, Edward Clarke, and Theodore Groves. Vickrey (1961) introduced the famous Vickrey auction (second price sealed bid auction). Clarke (1971) and Groves (1973) came up with a generalization of the Vickrey mechanisms and helped define a broad class of dominant strategy incentive compatible mechanisms in the quasilinear environment. For our model, the mechanism can be described as follows:

1. Each stakeholder i makes an announcement $\tilde{v}_i(x)$ for all suppliers x.

2. Based on the stakeholder's report-vector $\tilde{v}$, the mechanism calculates $x^* = x^{opt}(\tilde{v})$ as above (formula (2)).

3. It pays, to each stakeholder i, a sum of money equal to the total announcements for x^* of the other stakeholders:

$$p_i \coloneqq \sum_{j \neq i} \tilde{v}_j(x^*) \qquad (5)$$

4. It pays, to each stakeholder i, an additional sum, based on an arbitrary function of the announcements of the other stakeholders $h_i(\tilde{v}_{-i})$ so that the final payment t_i is defined as follows:

$$t_i = p_i(x^*) + h_i(\tilde{v}_{-i}) \qquad (6)$$

where $\tilde{v}_{-i} = (\tilde{v}_1, \dots, \tilde{v}_{i-1}, \tilde{v}_{i+1}, \dots, \tilde{v}_n)$, that is, h_i is a function that depends only on the announcements of the other stakeholders. Note that h_i may depend on $\tilde{v}_j(x), j \neq i$ of any x, but h_i must not depend on the choice of x^*.

This supplier selection mechanism is incentive compatible ("truth telling") in the sense of that announcing $\tilde{v}_i(x) = v_i(x)$ on every supplier x is an equilibrium[3] between all stakeholders i. For the proof, we have to show that there is no incentive for stakeholder i to announce more or less than the true valuation $v_i(x)$ as $\tilde{v}_i(x)$ on every supplier x. Let's assume stakeholder

[3] For any stakeholder i, the announcement of $\tilde{v}_i(x) = v_i(x)$ can only be the "truth" as long as i knows the truth herself. If there is an "error" between her assumption of $v_i(x)$ and a "real" $v_i(x)$, in the present paper we refer to her best knowledge of $v_i(x)$. In a sense, one should understand $v_i(x)$ as the true assessment of the real value of x for i, based on all the information i has at the time the announcement is to be made. The same holds finally for the total utility $u_i(x^{opt}) = v_i(x^{opt}) + t_i$.

i announces $\tilde{v}_i(x) = v_i(x)$ on every supplier x and let's first discuss deviations of $\tilde{v}_i(x^*)$. Like for a bidder in a second price auction, nothing would change if stakeholder i had announced $\tilde{v}_i(x^*) > v_i(x^*)$, because the payment t_i does not depend on her announcement and x^* will stay the choice anyway. But if she had announced less than $v_i(x^*)$ it could be that another x, say x', becomes the choice and thus she looses $u_i(x^*) = v_i(x^*) + t_i(x^*)$ (note that t_i depends on x^*) and she wins $u_i(x') = v_i(x') + t_i(x')$. But from the fact that $t_i(x) = p_i(x) + h_i$ where h_i does not depend on the chosen x follows $v_i(x') + t_i(x') < v_i(x^*) + t_i(x^*)$, because otherwise x^* would not had been the choice when $\tilde{v}_i(x) = v_i(x)$ for all x according to formula (2). Therefore, announcing less than the truth is a risk without advantage here for stakeholder i. Let's now check deviations of $\tilde{v}_i(x)$ for $x \neq x^*$. Here again it is exactly like for a bidder in a second price auction, that enhancing the announcement $\tilde{v}_i(x)$ doesn't gain any advantage but creates the risk that x becomes the choice, loosing $u_i(x^*) = v_i(x^*) + t_i(x^*)$ and gaining $u_i(x) = v_i(x) + t_i(x)$. But exactly as above for x', again $v_i(x) + t_i(x) < v_i(x^*) + t_i(x^*)$ shows that this is a deterioration and thus, in total, announcing $\tilde{v}_i(x) = v_i(x)$ is an equilibrium for all stakeholders i.

Note that for our purpose and our little proof, we don't use any condition like $\tilde{v}_i(x) > 0$ which is discussed when Vickrey-Clarke-Groves mechanism is applied to auctions. For more general results about the classical Vickrey-Clarke-Groves mechanism and supposedly more elegant proofs see, for example, Green and Laffont (1979).

5.1. Clarke pivotal rule implementation

For the practical implementation we still have to define which function h_i to use and we like to discuss the most popular function in the literature which is the Clarke pivotal rule function h^{CPR}. It's defined as follows (Clarke 1971):

$$h_i(\tilde{v}_{-i}) = h_i^{CPR}(\tilde{v}_{-i}) := -\max_{x \in X} \textstyle\sum_{j \neq i} \tilde{v}_j(x) \tag{7}$$

The total amount $-t = -(p + h)$ paid by the stakeholder i is then:

$$-t_i = -h_i(\tilde{v}_{-i}) - p_i = \max_{x \in X} \sum_{j \neq i} \tilde{v}_j(x) - \sum_{j \neq i} \tilde{v}_j(x^*) > 0 \tag{8}$$

where $-h_i(\tilde{v}_{-i})$ is the social welfare of others if i were absent and p_i is the social welfare of others when i is present.

As a result, only pivotal stakeholders – this are those stakeholders that have an effect to the supplier decision with their announcement – are paying to the mechanism. The amount they pay is the difference of social welfare of others if they were absent or present to the mechanism.

The first important observation is that for the payments t_i based on the Clarke pivotal rule, an assumption of $\tilde{v}_i(x) > 0$ is without loss of generality as mentioned above. Consider to add the constant (positive or negative) value c to all values $\tilde{v}_k(x), x \in X$ for any stakeholder $k \neq i$. Then c adds exactly one times to both $-h_i(\tilde{v}_{-i})$ and p_i, and the payment t_i stays unchanged.

The second remarkable observation is that for every stakeholder i, the payment $-t_i$ she has to pay "to the mechanism" never exceeds $\tilde{v}_i(x^*)$. For the proof see formula (8): If $-t_i$ would exceed $\tilde{v}_i(x^*)$, x^* could never be the optimum according to formula (2). This means a stakeholder cannot lose utility by participating – as long as she sticks to announcing true valuations $\tilde{v} = v$ according to incentive compatibility – which is referred to in literature as the "individual rationality" of the mechanism for the stakeholders.

Finally, the Clarke pivotal rule automatically gives us the additional property of the mechanism not to need to pay anything to the participants – there are only negative transfers $t_i < 0$ (see formula (8)). In practice, the question arises as to who these payments should go to. To the CFO? Let's see what theory says. But before that, we want to have a brief look to some examples.

5.2. Examples

Consider the following simple example. There are two suppliers, A and B, and two stakeholders, CFO and production division. In this example, CFO cares only about the supplier price and privately knows $q_A = 100$ and $q_B = 110$. The production division cares about some other specific supplier characteristics and evaluates possible savings from working with supplier B as 20. The resulting table represents values v for both stakeholders.

Supplier	A	B
CFO	10	0
Production	0	20

The direct calculations show that supplier B wins the demand and the resulting Clarke pivotal rule payments are, assuming that all players make truthful announcements $\tilde{v} = v$:

$$t_{CFO} = p_{CFO} + h_{CFO}(\tilde{v}_{Prod}) = 20 - 20 = 0$$

$$t_{Prod} = p_{Prod} + h_{Prod}(\tilde{v}_{CFO}) = 0 - 10 = -10$$

The resulting utilities are

$$u_{CFO} = v_{CFO} + t_{CFO} = 0 - 0 = 0$$

$$u_{Prod} = v_{Prod} + t_{Prod} = 20 - 10 = 10$$

Assume the similar situation with the only difference $q_B = 130$. Then:

Supplier	A	B
CFO	30	0
Production	0	20

In this situation the supplier A wins the demand and the resulting Clarke pivotal rule payments are

$$t_{CFO} = p_{CFO} + h_{CFO}(\tilde{v}_{Prod}) = 0 - 20 = -20$$

$$t_{Prod} = p_{Prod} + h_{Prod}(\tilde{v}_{CFO}) = 30 - 30 = 0$$

The resulting utilities are

$$u_{CFO} = v_{CFO} + t_{CFO} = 30 - 20 = 10$$

$$u_{Prod} = v_{Prod} + t_{Prod} = 0$$

Consider now the example with Production and Logistics departments and the following evaluations:

Supplier	A	B
CFO	10	0
Production	0	20
Logistics	15	0

In this situation the supplier A wins the demand and the resulting Clarke pivotal rule payments are

$$t_{CFO} = p_{CFO} + h_{CFO}(\tilde{v}_{-CFO}) = 15 - 20 = -5$$

$$t_{Prod} = p_{Prod} + h_{Prod}(\tilde{v}_{-Prod}) = 25 - 25 = 0$$

$$t_{Logis} = p_{Logis} + h_{Logis}(\tilde{v}_{-Logis}) = 10 - 20 = -10$$

The resulting utilities are

$$u_{CFO} = v_{CFO} + t_{CFO} = 10 - 5 = 5$$

$$u_{Prod} = v_{Prod} + t_{Prod} = 0$$

$$u_{Logis} = v_{Logis} + t_{Logis} = 15 - 10 = 5$$

Finally let's have a look on an example with three suppliers and four stakeholders:

Supplier	A	B	C	p	h	t	u
CFO	20	10	0	17	-25	-8	2
Production	0	5	10	22	-22	0	5
Logistics	2	0	7	27	-27	0	0
Quality	0	12	8	15	-22	-7	5

For all examples, the payment of the stakeholders equal the difference of sum of all other stakeholders' evaluation of the winning supplier, if the concerned stakeholder were absent or present (formula (8)) and the final utility is positive for all stakeholders.

For the last example let's demonstrate what happens if Boni are shifted by a constant value c for single stakeholders. Here, $c_{Production} = -5$, $c_{Logistics} = -2$ and $c_{Quality} = -8$:

Supplier	A	B	C	p	h	t	u
CFO	20	10	0	2	-10	-8	2
Production	-5	0	-5	12	-12	0	0
Logistics	0	-2	5	14	-14	0	-2
Quality	-8	4	0	8	-15	-7	-3

Note that the column of payments t doesn't change and the final utility u becomes negative, but shifted exactly by the values c_i. Finally, utility can only be interpreted relatively which means here, the only important thing for individual rationality is that participating doesn't get worse than the worst case without participating. For Logistics, the worst case $u = -2$ takes place with $x^* = B$, but for Quality, $u_{Quality} = v_{Quality}(x^*) + t = -3$ which is 5 better than the worst case – independent from $c_{Quality}$.

5.3. Budget balanced mechanism with known supplier prices

In this section we want to answer the question of who should get the payments of stakeholders in our incentive compatible mechanism of supplier selection and apply the following “balanced budget” theorem (Green and Laffont 1979):

Theorem 5.1 *(A Possibility Result for Budget Balance of Groves Mechanisms) If there is at least one agent (stakeholder) whose preferences are known (...) then it is possible to choose the functions $h_i(v)$ so that $\sum_{i=1}^{n} t_i(v) = 0$.*

In order to fulfill the conditions of the Theorem, we assume that the supplier prices are common knowledge for all stakeholders. Then the evaluations v_{CFO} are known in advance by all stakeholders and there is no need to create an incentive for this stakeholder to tell the truth – he does not have any private knowledge. So the final payment for this stakeholder can be used to balance the budget. We can define the function $h_i(v)$ as follows:

$$h_i(v) = \begin{cases} h_i^{CPR}(v_{-i}) & \text{if } i \neq CFO \\ -p_{CFO} - \sum_{i \neq CFO} t_i(v) & \text{if } i = CFO \end{cases} \quad (9)$$

It is obvious that $\sum_{i=1}^{n} t_i(v) = 0$. The incentive compatibility of this mechanism for all stakeholders $\neq CFO$ is also obvious, because for them it makes no difference whether the CFO receives the payments or any virtual account.

6. Conclusion: Finally, a case for practical VCG implementation?

Michael H. Rothkopf (2007) claims that, even if theoretically very appealing the truth-revealing ideal of VCG process is, in practice, usually an unreachable mirage: Vickrey auctions were rare in 1990 and remain so, and as far as I know, no one has conducted a general VCG process (i.e., not just a simple Vickrey auction or a market-clearing price auction of identical items) in real commerce.”

There is a list of the reasons Rothkopf discusses to give a reason for the lack of practical implementation.

Some reasons are related to the calculation complexity of the practical implementation in the general environment: the exponential growth of effort related to bid preparation and bid communication and the NP completeness of the winner determination problem. These considerations are usually relevant for auctions – but we consider a specific form of voting mechanism with a relative small number of bidders and much more simple decision problem.

Because the decision happens inside the same company, we can disregard problems related to capital limited bidders, problems associated with the disclosure of valuable confidential information and the fact that the process can be revenue deficient. We can as well disregard possible bid preparation costs.

Rothkopf (2007) follows Ausubel and Milgrom (1979) by stressing problems associated with various kinds of cheating including: false bids by the bid taker, collusion by competing bidders and vulnerability to the use of multiple bidding identities by a single bidder. But given the internal company and repeated nature of the interaction, we can mostly disregard such behavior.

As a result, we hope to propose a rare practical case of VCG mechanism implementation.

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