Munich Social Science Review, New Series, vol. 8(2024)

How the “Auction Cube” Supports the Selection of Auction Designs in Industrial Procurement[1]

Gregor Berz
Institut für Angewandtes Mechanismus Design (IFAMD),
Kutaisi International University (KIU) and Center of Conflict Resolution (CCR)
gregor.berz@ifamd.de

Florian Rupp
Institut für Angewandtes Mechanismus Design (IFAMD),
Kutaisi International University (KIU) and Center of Conflict Resolution (CCR)
Florian.Rupp@kiu.edu.ge

Brian Sieben
Hilti Aktiengesellschaft
brian.sieben@hilti.com

Abstract: This paper proposes a novel method to guide users in selecting the most appropriate combination of basic auction forms for single lot events, taking into account both market and supplier criteria. Assuming a normal distribution for the bidders’ indifference prices, their strategic margin is benchmarked with a threshold derived from the standard deviation σ of the bidders‘ identical distribution. This delivers a basic criterion for the trade-off between first-price and second-price auctions. Following Milgrom’s seminal approach, a reverse interpretation of the revenue equivalence theorem yields more differentiated recommendations. The proposed method allows users to quickly determine the most suitable auction design, leveraging the tremendous commercial potential of reverse auctions. To the best of the authors‘ knowledge, this is the first paper to propose a robust method to guide

[1]An earlier, German version of this text was published as pre-print May 3rd 2021 on https://arxiv.org. The authors would like to thank Kenneth Binmore for helpful comments, in particular about the origin of what we call a ‘Hongkong Auction’

.

ISSN 0170-2521
ISBN 978-3-88278-317-9
www.accedoverlag.de

practitioners in selecting the right combination of basic auction types, considering both market and commercial criteria. The results are obtained using order statistics.

1. Introduction

Reverse auctions have been established in industrial procurement for at least 20 years. In such auctions, suppliers bid for orders placed by the customer's procurement department. Yet the choice of the most appropriate auction procedure – depending on the prevailing market and competitive situation – still poses a challenge for procurement. Theory offers a set of basic auction types that can be combined in exclusion rounds. Using three criteria – 'number of bidders', 'dispersion of the bids', and 'risk aversion of the bidders' –, this paper derives a unique auction procedure that is combined from the basic auction types for each sort of decision situation with a single object of negotiation. In doing so, it follows the lead of Paul Milgrom (2004: 77): "One important use of the revenue equivalence theorem is as a benchmark for analyzing cases when the assumptions of the theorem do not hold." Specifically, we employ a reverse interpretation of the revenue equivalence theorem of auction theory.

The basic auction types that we consider are the most relevant ones in industrial goods markets (Berz (2014: 42 and 57)): first-price sealed-bid auction, second-price sealed-bid auction, English auction (ticker/dynamic), Hongkong auction, and Dutch auction.

Section 2 compares first-price and second-price auctions from the procurement perspective. Regarding the bidders' indifference prices, we assume the normal distribution $N(0, \sigma)$ with mean 0 and standard deviation σ. Regarding the bidders' strategic margin M in a first price auction, which is to be expected due to their risk aversion – which we assume to be identical in the way that the bidders claim a common strategic margin M–, we state a threshold depending on σ and the number of bidders n up to which a first-price auction yields lower expected revenue than a second-price auction. We use $2\sqrt{3}/(n+1) \cdot \sigma$ as a first approximation of this threshold before examining the relative error of this expression: In exact calculations for 28 different values of σ, keeping the number of bidders n constant, we obtain the ever-same relative error. This error is indeed independent of σ because the expected value $E(X, i)$ of the i^{th} order statistic of the normal distribution $N(0, \sigma)$ with a fixed sample size n is already proportional to σ anyway. This means that for every n there is a factor $g(n)$ such that $g(n) \cdot \sigma$ constitutes the exact threshold. The appendix presents the values of $g(n)$ for 2 to 100 bidders calculated at ten decimal places.

Section 3 discusses the circumstances under which it is generally advisable to stage an auction in a given decision situation. Among these criteria are the risk of bidder's implicit collusion and asymmetric buyer preferences regarding the bidders. Finally, we state three conditions that must be met for the "Auction Cube" introduced in Section 5 to be applicable.

In Section 4, we discuss the expected results of the basic auction types, depending on σ, n and M. Furthermore, we analyse properties of these auction types gleaned from many years of industry experience, including bidder collusion and the "winner's curse." Under the assumption that the criteria stated in Section 3 are met, i.e., that the conditions favor an auction, Section 5 finally introduces the "Auction Cube", an instrument to conveniently determine the optimal auction type for each set of circumstances. Each vertex of the cube represents one of the eight constellations – (n low, n high) x (M low, M high) x (σ low, σ high) – and recommends a suitable compound auction procedure for practice.

2. First-price versus second-price auctions

We consider a decision situation with $n > 1$ bidders, whose indifference prices for the object of negotiation are identically and normally distributed with a standard deviation of σ. Without loss of generality, the expected value is normalised to 0. The indifference price is that value which makes a bidder indifferent as to whether or not he sells the good. The well-known truth-revealing quality of a second-price auction (which was already exploited by Johann Wolfgang von Goethe in the sale of a manuscript in 1797, see Holler and Klose-Ullmann (2020: 239)) – namely that a bidder's offer will only affect her chances of winning but not the price she receives if she wins – ensures that offering the indifference price is the strictly dominant strategy for all bidders. Thus, the expected result of a second-price auction, $E(\text{SP})$ ("SP" for second-price auction), equals the expected value of the second order statistic $X(2, n)$ with sample size n of the normal distribution $N(0, \sigma)$.

$$E(SP) = E(X(2, n)) \qquad (1)$$

In a first-price auction, by contrast, none of the same bidders can be assumed to offer their indifference price. For if a bidder were to offer her indifference price and thereby win the auction, then – per definition of the indifference price – her profit would be zero. Therefore, every bidder will add a strategic margin M (Berz (2014: 77f)) to her indifference price. The size of M is inversely related to her aversion to the risk of not winning the auction. Thus, assuming that the risk aversion of all bidders is equal in the way that the suspected strategic margin M in a first price auction is equal[2], the expected result of a first-price auction $E(\mathrm{FP})$ ("FP" for first-price auction) equals the sum of M and the expected value of the first order statistic $X(1, n)$ with sample size n of the normal distribution $N(0, \sigma)$.

$$E(FP) = E(X(1, n)) + M \qquad (2)$$

Thus, the buyer can expect to achieve a better result with a first-price auction than with a second-price auction exactly if

$$M < E(X(2, n)) - E(X(1, n)) =: G \qquad (3)$$

Sadly, the expected values of the order statistics of the normal distribution cannot be determined precisely. While concrete values can be calculated as integrals of exponential functions at arbitrary precision, there is no closed formula in σ and n for these values. This may explain why the game theory literature often assumes uniform distributions for the bidders' valuations. The i^{th} order statistics $Y(i, n)$ of the uniform distribution over $[0,1]$ are the beta distributions $B(i, n - i + 1)$ with the expected values $E(Y(i, n)) = i/(n + 1)$, as is easy to appreciate by applying the formula for $f_{X(i,n)}$ from David (2003:10f), reproduced in the appendix, to the simple case of the uniform distribution. The obtained formulas for $f_{Y(i,n)}$ and $E(Y(i, n))$ are easy to calculate and argue with.

Yet the assumption of uniform distribution entails a major drawback: Even the "continuous" uniform distribution over $(0,1)$ becomes discontinuous at least at the limits of the interval, making it a probability distribution that cannot be observed in practice. The question as to which values should be assumed as the lower and upper interval limits of the

---

[2] This implicates, that the identical probability distribution of the indifference prices of the bidders and especially it's expectation value must not be assumed to be common information here, because otherwise *M* depends on the private realization of each bidder's indifference price.

uniform distribution can regularly not be answered in practice. To transform the closed interval $[0,1]$ into the open interval $(0,1)$ in order to preserve its "continuity" amounts to a mathematical artifice and is ultimately but a cosmetic fix. By no means can the interval limits 0 and 1 be assumed "without loss of generality" because the uniform distribution over $(0,1)$ always has the same standard deviation $1/(2\sqrt{3})$. In practice, however, we can observe bidders with different dispersions of their indifference prices. This cannot be represented using the normalising assumption of the compact interval $(0,1)$ that is so common in auction theory.

To approximate the expected values of the first and second order statistic of the normal distribution, we draw on the assumption that the order statistics Y' of a uniform distribution with expected value 0 and standard deviation σ behave similarly to those of the normal distribution with the same parameters.

For the order statistics Y'' of the uniform distribution over $(-1/2,1/2)$ with standard deviation $\sigma''= 1/(2\sqrt{3})$, we obtain $E(Y''(1,n)) = 1/(n+1) - 1/2 = (1-n)/(2n+2)$ and $E(Y''(2,n)) = 2/(n+1) - 1/2 = (3-n)/(2n+2)$. Scaling by $2\sqrt{3}\cdot\sigma$ to a uniform distribution over $(-\sqrt{3}\cdot\sigma, \sqrt{3}\cdot\sigma)$ with standard deviation σ yields $E(Y'(1,n)) = 2\sqrt{3}\,(1-n)/(2n+2)\cdot\sigma$ and $E(Y'(2,n)) = 2\sqrt{3}\,(3-n)/(2n+2)\cdot\sigma$.

This allows us, as a first approximation, to state a threshold G' for M below which a first-price auction can be expected to yield a better result for the buyer than a second-price auction:

$$M < E(Y'(2,n)) - E(Y'(1,n)) = \frac{2\sqrt{3}}{n+1}\cdot\sigma =: G' \quad (4)$$

This result permits a reverse interpretation of the revenue equivalence theorem (Milgrom 2004: Section 3.3.4, and Klemperer 2004: Section 1.4), a core achievement of auction theory. Besides other conditions, the theorem assumes risk-neutral bidders with indifference prices that are manifestations of an identical continuous probability distribution. In the reverse interpretation, the theorem thus says that, given the assumed conditions, the strategic margin to be expected[3] of a risk-neutral bidder is exactly $M = G' =$

[3] Here, we more precisely mean a mean of *M* because *M* can't be assumed to be equal for all bidders after one assumption of the revenue equivalence theorem is that the probability distributions of their indifference prices, including the expectation value of this distribution, is common information for all bidders

$2\sqrt{3}/(n+1)\cdot\sigma$. Or, if we assume the uniform distribution over the interval $(0,1)$ in line with the common normalisation in the auction theory literature, we obtain $\sigma = 1/(2\sqrt{3})$ and thus $M = 1/(n+1)$ for the expected strategic margin of a risk-neutral bidder. The latter also exactly equals the difference between the expected values of the first and the second order statistic of the uniform distribution over $(0,1)$. Yet to apply this result to a group of bidders with any σ observed in practice, the interval (a,b), across which the indifference prices are assumed to be uniformly distributed, must be suitably chosen such that $\sigma = (b-a)/(2\sqrt{3})$ – this corresponds exactly to the approach described above. The size of σ is inherent to the chosen interval width $b-a$ of the assumed uniform distribution.

If, in order to ensure an unbounded continuous probability distribution, we instead consider the bidders' indifference prices to be normally distributed with standard deviation σ, what is the error produced by the approximation through $G' = 2\sqrt{3}/(n+1)\cdot\sigma$ as the limit for M? To evaluate this error, we used the Maple software to calculate, to ten significant decimal places, the exact values of G, i.e. $E(X(1,n))$ and $E(X(2,n))$, for all $n = 2,3,4,5$ and 6, as well as for all $\sigma = 0.1, 0.2, \dots, 1, 2, \dots, 10, 20, \dots, 100$. We find that, for each n, the relative error $(G' - G)/G$ is exactly the same for all σ. In fact, $E(X(i,n))$ is already proportional to σ, as can easily be deduced[4] from the scaling property of the normal distribution $N(0,\sigma) = N(0,1)\cdot\sigma$. Thus, for every n there is a constant value $g(n)$ for which, independently of σ, the following applies regarding the threshold G of the strategic margin M:

$$G = E(X(2,n)) - E(X(1,n)) = g(n)\cdot\sigma \qquad (5)$$

The appendix lists all these factors $g(n) = G/\sigma$ for $1 < n < 101$, all calculated with $\sigma = 1$.

[4] This may seem surprising if the density functions $f_{X(i,n)}$ of the order statistics of the normal distribution and their expected values are considered as integrals: $E(X(i,n)) = \int_{-\infty}^{\infty} t\cdot f_{X(i,n)}(t)\,dt$. However, the realisation of the normal distribution $N(0,\sigma)$ can also be understood as the multiplication of σ with the random values of a realisation of the normal distribution $N(0,1)$. This immediately shows that the scaling by σ extrudes through the realisations of the order statistics and their expected values.

Examples:

Let us assume that, in a first offer round, four bidders have submitted the prices $P_1 = 112.18$, $P_2 = 95.75$, $P_3 = 109.27$ and $P_4 = 94.15$. Then the best approximation of σ that is available at the time is the sample variance $\sqrt{\frac{1}{4}\Sigma_{i=1}^{4}(P_i - \frac{1}{4}\Sigma_{j=1}^{4}P_j)^2)} = 7.97440711$.

We furthermore obtain $G' = 2\sqrt{3}/(n+1) \cdot \sigma = 5.52483131$. Thus, if the bidders' strategic margins are below $G' = 5.52$, a first-price auction will give the buyer better results in terms of expected values. Otherwise, a second-price auction is to be preferred.

Under the arguably more realistic assumption that the bidders' valuations are normally distributed, we use the value $g(4) = 0.732363991$ from the appendix to calculate G as $g(n) \cdot \sigma = 5.840168614$. In this scenario, the buyer can expect the result of a first-price auction to exceed that of a second-price auction if the bidders' strategic margins do not exceed $G = 5.84$.

Let us now assume that a fifth bidder submits a price of $P_5 = 97{,}33$. This yields a new estimate of σ (7.200248385) and a new G' (4,157065343). For G, we use $g(5) = 0.667945504$ to obtain $G = 4.809373533$.

Finally, let there again be four bidders, but their first-round offers are much less dispersed: $P_1 = 100.73$, $P_2 = 100.86$, $P_3 = 99.05$ und $P_4 = 100.12$. For our estimate of σ, we obtain the much smaller sample variance of $0{,}715017482$. In this second example, we have $G' = 0.495378643$ and $G = 0.523653057$.

This tells us that the bidders' maximum strategic margins that still make a first-price auction preferable to a second-price auction from the buyer's point of view significantly depend on σ. The sample variance of the prices that are already available before the auction can provide a very rough indication of the same bidders' σ. Regarding the expected strategic margins, these available prices have to be compared to the result of a cost structure analysis, which can provide insights regarding the indifference prices.

3. Practical precondition for auctions

The most important and widely observed impediment to a successful auction is implicit collusion among the bidders that is not outspoken and therefore off the radar of competition law. Bidders who would rather retain their established margin in the market than undercut each other's prices tend to bid less aggressively and are quite prepared to leave a chunk of business to a competitor. The game theory literature refers to such behavior as "strategic supply reduction", since it is typically associated with a reduction in the quantities offered (cf., e.g., Klemperer (2004: 64)) on collusive markets. Markets are likely to be implicit collusive if, although in sum there is excess supply, few suppliers face many buyers. Then coordination among the suppliers may allow them to turn the tables, i.e. to "simulate" a seller's market in what, based on the excess supply, would otherwise be a buyer's market. Often such coordination does not even require explicit agreement; price transparency, e.g., in the form of a public index, can suffice as a coordination mechanism. By increasing price transparency, an auction can even exacerbate the problem of implicit collusion in such a situation. Other negotiation and decision mechanisms that are specifically tailored to reduce the effects of implicit collusion will then produce better results. In sum, to keep matters simple, we just state "no bidder collusion" (neither implicit nor explicit collusion) as the most important precondition for a successful auction. A more detailed investigation of implicit collusion ought to consider at least the following additional criteria:

- number of suppliers
- number of buyers
- frequency of the buyer's decisions regarding suppliers
- general degree of price transparency
- barriers to market entry
- historical relations among the market participants

The second precondition for an auction to yield the best expected result in a procurement decision is the buyer's absolute price sensitivity. Only if any minimal price difference between two otherwise equal alternatives already suffices to make the buyer better off if he selects the cheaper alternative does an auction best satisfy his best preference for the best offer. To that end, the method of 'bonus systems' has been established in industrial practice for many years (Berz (2014:131ff)). Through a monetary evaluation of all the differences between the bidders and their offers that may be relevant to the decision – note that the analysis should cover all the contractual properties regarding the object of negotiation, as well as all the corporate properties of the bidders as potential contractual partners –, bonus systems accentuate the price discrimination among the offers and thus maximize the buyer's price sensitivity.

If we quite succinctly formulate "comparable bidders" as the second precondition for recommending an auction, by that we mean, firstly, that all different evaluations of the bidders are covered by that bonus system's price discrimination and that the buyer is then indeed maximally price sensitive. Secondly, we mean that all bidders can be assumed to have the same degree of aversion to the risk of not winning the contract. This is necessary so that in the decision between a first-price and a second-price auction we can assume the same strategic margin M for all bidders.

A final consideration refers to situations in which there are internal strategic reasons not to expose a certain particularly trusted supplier to any competition. This case calls upon cooperative game theory in order to identify a 50:50 split of the pie generated by the cooperation with that supplier. In terms of the question whether holding an auction is advisable at all in such a situation, a trusted supplier only constitutes a special case of a much more general situation, which is very common in practice: For internal reasons, the procurement department will only invite a very small number of suppliers to bid. Ultimately yet more general but equivalent for our purposes is the situation where only one or two suppliers are available in the market to submit offers. Whatever the reason for the scarcity of bids, no auction can be held with only a single bidder (leaving aside the special case of a first-price sealed-bid auction with a single bidder, which we do not discuss here). An auction with two bidders can only be recommended unconditionally if the bidders do not know that there are only two of them. Otherwise, the decision whether to hold an auction requires further analysis of the competitive situation between the two bidders.

In sum, we obtain the following preconditions for a successful auction:

- The market must be competitive – there must be no collusion (neither implicit nor explicit), and there must be at least two bidders (with exactly two bidders, they must think there are more of them).
- The bidders must be comparable – both in terms of their risk aversion and in terms of the buyer's, eventually asymmetric preferences regarding them.
- As a third, rather technical precondition for the application of the "Auction Cube" introduced in Section 5, the auction must be about a single object of negotiation, as opposed to multiple lots that may be won by different bidders.

4. Discussion of the basic auction types

First-price sealed-bid auction: In a first-price sealed-bid (“FPSB”) auction, all bidders submit exactly one bid and do not learn about the bids placed by the competition until the buyer makes his decision. The buyer sifts all the offers and awards the contract to the bidder with the best offer. That bidder receives the price stated in his own offer, which is why a FPSB auction is also referred to as a “pay-as-bid” or “pay-your-bid” auction (Klemperer (2004: 115)).

The expected result of a FPSB auction corresponds exactly to what we stated in Section 2 regarding general first-price auctions:

$$E(FPSB) = E(X(1,n)) + M \quad (6)$$

In practice, a FPSB auction can smack of information exposure: Having learned about the offers of all bidders, the buyer may be tempted to exploit this information as an argument to re-negotiate the result of the auction. Therefore, in FPSB auctions, the decision commitment of the buyer is particularly important. We will take that commitment to be ironclad for the purpose of this paper.

A further phenomenon of FPSB auctions that is both observed in practice and discussed at length in the game theory literature is the “winner’s curse” (Holler and Klose-Ullmann 2020: 249f). If the bidders are very uncertain regarding the valuation of the object of negotiation but fundamentally assign it the same value (a so-called “common value” situation), the winner of the auction will be the bidder who overvalued the object the most – seemingly to his benefit, but in actual fact to his detriment. Being awarded the contract, he learns of his misjudgment and must accept his own unfavorable price – hence the “winner’s curse”. The effect that risk-averse bidders bid cautiously in a FPSB auction in order to avoid the winner’s curse, as sometimes discussed, is only displayed by very experienced bidders in practice. If, by contrast, the other type of risk aversion prevails, namely the aversion to not winning the auction, the winner’s curse is quite common.

Second-price sealed-bid auction: In a second-price sealed-bid (SPSB) auction, too, the bidders submit exactly one offer each and learn about their competitor's bids only once the buyer makes his decision. While the best offer wins, the winner receives only the price offered by the second-best bidder. As already mentioned in Section 2, this yields the strictly dominant strategy for the bidders to offer exactly their indifference prices – the truth-revealing mechanism (Holler 2020: 239).

The expected result of a SPSB auction corresponds exactly to what we stated in Section 2 regarding general second-price auctions:

$$E(SPSB) = E(X(2, n)) \qquad (7)$$

In the practice of SPSB auctions, the risk of information exposure is even greater than in a FPSB auction because SPSB auctions in theory reveal the bidders' genuine indifference prices. SPSB auctions furthermore require considerable skill, trust, and integrity of all decision-makers involved: The buyer may feel a strong temptation to pay the winner only the price he bid, rather than the second-best price – and the bidders must be confident that this never happens.

The game theory literature depicts SPSB auctions as being particularly prone to collusion among the bidders (e.g., Klemperer (2004: Section 19)). If the bidders reach an agreement, by the design of their offers they cannot only control who wins and at what price, but they can moreover minimize the incentive for cartel members to defect – the chosen winner merely needs to offer a price that is actually unattractive for the bidders. This however requires a truly explicit agreement among the bidders, who are therefore at risk of conflict with competition law. If, however, the market exhibits only implicit collusive tendencies but no explicit cartel agreements, our experience does not confirm that a SPSB auction promotes implicit bidder coordination. On the contrary: If the buyer successfully applies strategic measures against implicit collusion – e.g., the bundling of all potential business with the bidder in the foreseeable future – in order to induce at least two bidders to defect, then the SPSB auction can completely liberate the submitted prices from any historically established margin of collusion.

English auction: In an English auction, the buyer informs all bidders about the other bidders' offers – strictly anonymously, so as not to violate competition law – and invites them to improve their bids until only one bidder remains: the winner of the auction. Several variants of the English auction, which appear quite different at first glance, have established themselves in the practice of industrial procurement: As an alternative to the disclosure of all offer prices, the information provided to the bidders can also be limited, comprising for example only the best bid, or each bidder's position in the ranking of bids, or a color code in which each color represents several raking positions. In addition to such "dynamic English auctions", where all bidders themselves actively submit and improve their offers, the English ticker auction has also become commonplace in practice. Here, the buyer actively calls stepwise descending prices and after each tick informs the bidders whether more than one bidder has confirmed the previous step – otherwise the auction is over. Alternatively, the buyer may also state the number of bidders left over after each step. This ticker variant of the English auction is also known in practice as a Japanese auction. Allegedly, this term goes back to Japanese fish markets although, as on the daily fish market in Hamburg, in that situation, a descending clock auction as a forward auction was the likely choice. Thus, the "Japanese" fish market auction is rather a Dutch auction (Berz (2014: 38ff)) and therefore a misnomer. The separate name is redundant anyway because the English ticker auction, at least in the theoretical case of infinitesimally small ticker steps, is the purest form of implementing the idea of the English auction.

At first glance, the result of an English auction in expected value terms equals that of a SPSB auction, i.e. $E(\text{English}) = E(X(2,n))$ (Berz 2014: 32). However, the transparency of the offer prices has the intended effect that, under the pressure of competition, the bidders readjust their indifference prices during the auction. This is particularly the case when the calculation of the indifference prices is subject to uncertainty and when, additionally, a common value situation applies. This means that the auction changes both the expected value and the σ of the normal distribution that is to be assumed regarding the indifference prices. Accordingly, by Z we denote the order statistics of the normal distribution obtained after the bidding process, which gives us the following expected result of the English auction:

$$E(English) = E(Z(2,n)) \qquad (8)$$

The price competition that is inherent to the bidding process typically reduces the σ of the normal distribution of the indifference prices – the bidders are driven closer together. Whether the expected value also declines, i.e., whether $E(Z(2,n)) < E(X(2,n))$, which would favour the buyer, depends on the bidder's risk aversion to the winner's curse that we already discussed above. The English auction can help those bidders avoid the winner's curse who are especially exposed to it because they are more averse to the risk of not winning the auction than to the risk of winning it on bad terms. The buyer, in turn, must be very clear about whether or not he wants to prevent the winner's curse. Preventing it may increase the expected result of the auction, $E(Z(2,n)) > E(X(2,n))$. This is precisely the reason why the practice of industrial procurement almost exclusively favours forms of the English auction in which the bidders do not know how many of the other bidders have already dropped out. This is to ensure $E(Z(2,n)) \leq E(X(2,n))$, yet it does not prevent the winner's curse. The only widely used auction form in industrial procurement practice that avoids the winner's curse is that type of English ticker auction in which the number of remaining bidders is counted down (Berz (2014: 93, 97)). Incidentally, the relation $E(Z(2,n)) \leq E(X(2,n))$ need not hold even without the information as to how many other bidders have already dropped out. For if due to the bidders' implicit collusive behaviour, which was mentioned in Section 3 and also discussed in the context of the SPSB auction in Section 4, the expected value of the normal distribution possibly remains the same while σ declines as the indifference prices move closer together, then $E(Z(2,n))$ will exceed $E(X(2,n))$. In practice, this is prevented by stipulating that once submitted, the prices cannot be raised. Then, however, no movement at all will arise in the auction in such a situation – an effect that can indeed be observed in practice ("auction fail").

Hongkong auction: The Hongkong auction proceeds just like the English ticker auction, the only difference being that in the former, the $m > 1$ best bidders are the winners, as opposed to $m - 1$ winners in the latter. The last bidder who exits the ticker and thereby triggers the termination criterion of only $m - 1$ bidders remaining learns only after his exit that he, too, is a winner. Therefore, the Hongkong auction cannot be counted down to its end; as soon as only $m + 1$ bidders remain active in the auction, all that the bidders can be told after each ticker step is whether more than $m - 1$ of them are still active.

The name of the Hongkong auction derives from its application in the sale of 3G mobile spectrum licenses in Hongkong (Klemperer 2004: Section 4.4.1). Put simply, m winners were to be identified among n bidders. Actually – even if not mentioned by Klemperer – it was Ken Binmore, who was in charge of the auction design of 3G in Hongkong and he and his team proposed, following an offer round with all n bidders, to select the m bidders with the m best prices, but with all being assigned the price that from the buyer's perspective takes the m^{th} rank. In other words, the Hongkong Auction is a "uniform price auction" with the price of the lowest-ranked winner applying to all winners. If we were to assume that the bidders' indifference prices do not yield to competitive pressure during the auction, the result of a uniform price auction would be exactly the same as that of a Hongkong auction being a descending ticker auction.

At first glance, the result of a Hongkong auction with m winners in terms of the obtained price per winner equals that of an English auction with $m - 1$ winners, i.e. $m \cdot E(Z(m,n))$ in sum. However, the Hongkong auction does not constitute a truth-revealing mechanism since the departing bidder still has a chance of winning and being paid the price that he offered. Therefore, similarly to a first-price or pay-as-bid auction with m winners, every bidder will add a strategic margin $N > 0$ to his offer[5] to preclude the case that if he turns out to be the "worst winner", he is actually left without any profit at all. It can furthermore be argued that N is strictly smaller than M because if a bidder wins, with probability $(m-1)/m$ it is not him but someone else who sets the uniform price that applies to all winners. While this does not yet trigger the sweeping truth-revealing effect, at least it provides an incentive to reduce the strategic margin contained in the offer below M. With this N, $0 < N < M$, we thus obtain:

$$E(Hongkong(m)) = m \cdot (E(Z(m,n)) + N) \qquad (9)$$

In analogy to Section 2, we can now approach a threshold H of the size of the strategic margin N up to which a Hongkong auction with m winners is preferable to an English auction with the same number of winners. In terms of the expected values of the respective order statistics, we obtain:

$$N < H := E(Z(m+1,n)) - E(Z(m,n)) \qquad (10)$$

If, as in Section 2, for the approximation H' of H we approximately assume the indifference prices to be uniformly distributed over $(-\sqrt{3}) \cdot \sigma, \sqrt{3} \cdot \sigma)$, instead of Z, we can again rely on the order statistics Y' of the uniform

[5] As for M, we assume for N that the bidders are equal in their risk aversion in the way that they all add the same strategic margin N in a Hongkong auction.

distribution, which we have recognised as beta functions. In analogy to Section 2, it is easy to derive that for all i with $0 < i < n + 1$, we have $E(Y'(i, n)) = 2\sqrt{3}\,(2i - 1 - n)/(2n + 2) \cdot \sigma$, and so we obtain:

$$H' = E(Y'(m+1, n)) - E(Y'(m, n)) =$$
$$= \frac{2\sqrt{3}\,(2(m+1) - 1 - n)}{2n+2} \cdot \sigma - \frac{2\sqrt{3}\,(2m - 1 - n)}{2n+2} \cdot \sigma =$$
$$= \frac{2\sqrt{3}}{n+1} \cdot \sigma = G' \qquad (11)$$

For $m > 1$, we may be interested in comparing the Hongkong auction not just to the English auction but also to a pay-as-bid auction with m winners. The latter has an expected result of $\Sigma_{i=1}^{m} E(X(i, n)) + m \cdot M$, or $E(X(1, n)) + E(X(1, n)) + 2 \cdot M$ for $m = 2$. For the decision whether to choose a Hongkong auction, it is insightful for the buyer to examine the expected values for $m = 2$ under the assumption of a uniform distribution:

$$E(English(m = 2)) = 2E(Y'(3, n)) = \frac{2\sqrt{3}\,(5-n)}{n+1} \cdot \sigma$$

$$E(Hongkong(m = 2)) = 2(E(Y'(2, n)) + N) = \frac{2\sqrt{3}\,(3-n)}{n+1} \cdot \sigma + 2 \cdot N$$

$$E(pay - as - bid(m = 2)) = E(Y'(1, n)) + E(Y'(2, n)) + 2 \cdot M =$$

$$= 2\sqrt{3} \cdot (\frac{1-n}{2n+2} + \frac{3-n}{2n+2}) \cdot \sigma + 2 \cdot M = \frac{2\sqrt{3}\,(2-n)}{n+1} \cdot \sigma + 2 \cdot M$$

For example, for $n = 6$ and $\sigma = 1/(2\sqrt{3})$, we obtain:

$$E(English) = -\frac{1}{7}$$

$$E(Hongkong) = -\frac{3}{7} + 2 \cdot N$$

$$E(pay - as - bid) = -\frac{4}{7} + 2 \cdot M$$

To favor a Hongkong auction over an English auction, $N < 1/7 = G'$ must apply. To favour a pay-as-bid auction over an English auction, $M < 1/14$

must apply. And finally, to favour a Hongkong auction over a pay-as-bid auction, $N < M - 1/14$ must apply.

For illustration, Figure 1 shows the average auction result achieved per winner over the interval $(-1/2, 1/2)$.

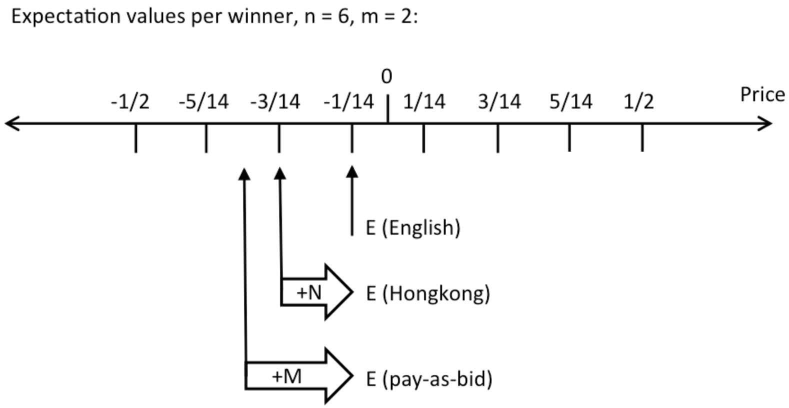


Figure 1: Comparison of the expected results of the basic auction types

Using the formula we already mentioned, $E(Y'(i,n)) = 2\sqrt{3}\,(2i - 1 - n)/(2n + 2) \cdot \sigma$, and assuming a uniform distribution, these reflections can easily be extended to $m > 2$. However, we will address the case that is arguably more relevant in practice – normally distributed indifference prices – and focus on $m = 2$ since in Section 5 we will use the Hongkong auction solely as a first round to establish two winners, who subsequently compete with each other in a final round.

Calculating the exact values for $H = E(Z(3,n)) - E(Z(2,n))$ using the Maple software, we notice – as we already did in Section 2 regarding the first and second order statistic – that for the third order statistic, too, with a normal distribution and standard deviation σ, the expected value is proportional to σ. Again, this result follows immediately from the scaling property of the normal distribution (cf. Section 2). The factors $h(n)$ to calculate $H = h(n) \cdot \sigma$ are listed in the appendix for $n = 3, \ldots, 100$.

Dutch Auction: In a Dutch auction, the auctioneer actively calls prices in ascending steps and after each step informs the participants whether a bidder has confirmed that step; if that is so, the auction is over. In analogy to the English ticker auction, the theoretically purest implementation of the Dutch auction is to proceed in infinitesimally small ticker steps. The starting price of a Dutch auction can be arbitrarily low.

As a first approximation, the expected result of a Dutch auction equals that of a first-price auction:

$$E(Dutch) = E(FP) = E(X(1,n)) + M \qquad (12)$$

In practice, however, two important effects tend to reduce M. Both effects are larger the stronger the bidders' aversion to the risk of not winning the auction.

Firstly, the Dutch auction does not feature the sort of information exposure that we discussed in the context of the FPSB auction. The moment a bidder confirms a ticker step, he can be sure to receive the contract. Thus, he need not worry about revealing his indifference price and subsequently not receiving the contract after all. This property of the Dutch auction may also be interpreted as “dual transparency”. Whereas in the English auction, which thrives on price transparency, the bidders always know that some competitors are still going along with the price, the exact opposite applies to the Dutch auction: So long as the ticker steps are not confirmed, all bidders know that *no* competitor goes along with these prices.

The second reason why bidders in a Dutch auction bid with lower strategic margins than in a FPSB auction is that in the former, the decision commitment of the buyer is beyond dispute – he only receives a price from a single bidder in the first place. The Dutch auction therefore does not produce an argument with which the buyer might approach the winner about a re-negotiation of the result. Knowing this, all bidders can be sure that the winner of a Dutch auction indeed receives the business, with no room for re-negotiation. Therefore, the bidders have no need for any risk margins or “buffers” that they might build into their offer prices or, more specifically, into their strategic margin, to provide for the risk of re-negotiation.

5. The Auction Cube

Assuming that the preconditions for a successful auction that we stated in Section 3 are met, and building upon the basic auction types presented in Section 4, we now identify those (compound) auction designs that promise the best expected result for the buyer, depending on three factors: the number of bidders n; the bidders' strategic margin M in a first-price auction, which is inversely related to their aversion to the risk of not winning the auction; and the standard deviation of the bidders' indifference prices σ (assuming an identical normal distribution for all bidders).

These three factors n, M and σ generate a three-dimensional cube, and for practical purposes we assume that each dimension can assume only one of two values: "high" and "low". Each of these distinctions is to be interpreted in such a manner that the inequalities regarding the decision between a first-price and a second-price auction that we stated in Section 2 are either clearly satisfied (in some corners of the cube) or indifferent (in others). Regarding n, it suffices to conceive of "high" as $n > 3$ in order to meaningfully apply an English or Hongkong auction. The Auction Cube does not prejudice the decision whether the winner's curse is to be avoided, i.e. whether the number of the remaining bidders is counted down. This decision can still be taken in the relevant corners of the cube.

Origin of all thoughts regarding the auction designs in the corners of the Auction Cube is the so-called "Klemperer auction", which the inventor himself refers to as an "Anglo-Dutch auction" (Klemperer 2004: 116). If no sufficient information is available regarding the bidders' competitive situation, i.e., their indifference prices and degree of risk aversion, according to Klemperer, this auction type is especially "robust" and therefore recommended in case of doubt. Ultimately, the idea is to influence the indifference prices by means of price transparency in a first, "English" round so that, in the terminology of Section 3, we may calculate with the order statistics Z rather than X. The remaining bidders are then invited to a final Dutch auction to exploit the lowest indifference price; however, a strategic margin may have to be accepted.

In the industrial procurement practice, an adaptation of the Klemperer auction has been established for many years: a Hongkong auction with two winners, followed by a Dutch auction among those two bidders. This Hongkong–Dutch design does justice to the experience that often only a handful of bidders are available, and the offer prices begin to rise steeply already after the second-best bidder. In the terminology and in accordance with the reflections of Section 4, this means that in practice, the bidders are usually assumed to have a strategic margin N in a Honkong Auction that is below the threshold H below which a buyer should prefer a Hongkong auction to an English auction with two winners.

This Hongkong–Dutch auction design is indeed what the Auction Cube recommends in two of its eight corners. In the other six corners, due to the circumstances that prevail in each situation, further adaptations are called for. In the following, we discuss each of the eight scenarios in detail. Figure

2 at the end of the section summarizes this exposition graphically in the form of the Auction Cube.

Constellation $n > 3$*, low risk aversion, low* σ*:*

In the constellation of a sufficient number of bidders who are not overly averse to not winning the auction and whose indifference prices are close together, all the aspects of auction theory that we discussed in detail in Sections 2, 3 und 4 speak in favor of an English auction, ideally as a ticker that runs to the end, when the winner has been determined.

→ Recommended auction design: *English auction*

Constellation $n > 3$*, low risk aversion, high* σ*:*

The more widely the indifference prices are assumed to be dispersed, the higher the threshold $G = g(n) \cdot \sigma$ of M that we discussed in Section 2, up to which a first-price auction delivers a superior final price. Quite in accordance with the practical experience mentioned above, in this situation we recommend a Hongkong auction to select two participants for the final decision round. This final round should be a first-price auction to exploit the price of the best bidder, while accepting his strategic margin. Auction theory is largely indifferent as to whether the second round should take the form of a Dutch auction or a FPSB auction. As discussed in Section 4, we can only expect a reduced strategic margin in a Dutch auction if the bidders are sufficiently keen to win. Conversely, with a low degree of risk aversion, the mere announcement of a Dutch auction can provoke defensive reactions from some suppliers, or even their non-participation in the auction. We therefore recommend a FPSB auction.

→ Recommended auction design: *Hongkong – FPSB*

Constellation $n > 3$*, high risk aversion, high* σ*:*

If the bidders are highly averse to losing in the auction, small strategic margins are to be expected, and a first-price auction is to be recommended for the final round in any event. Given enough bidders, the first round can take the form of a Honk Kong ticker auction, with the aim of influencing the indifference prices, as discussed in Section 4. If the bidders are sufficiently risk averse, a Dutch-style final round can serve to reduce the strategic margin.

→ Recommended auction design: *Hongkong – Dutch*

Constellation $n > 3$, high risk aversion, low σ:

With many bidders and high risk aversion, the dispersion of the indifference prices plays no role. Even if, with low M and low σ, the inequality $M < g(n) \cdot \sigma$ is questionable, then both sides of it are small, and therefore so is any disadvantage that a first-price auction might entail.

→ Recommended auction design: *Hongkong – Dutch*

Constellation $n = 2$ or 3, low risk aversion, low σ:

If there are not enough bidders for a Hongkong ticker and low risk aversion furthermore speaks against a Dutch auction (the advantages of a Dutch auction are reduced without risk aversion of the bidders and they might rather tend not to accept the process), we recommend two rounds of FPSB auctions. The first of them is actually not a first-price auction in the strict sense but rather a pay-as-bid auction with two winners, yet we refer to it as FPSB to keep the terminology simple. Both rounds are sealed-bid with ex-post transparency: the bidders are informed of the (anonymised) offers of all other bidders after the decision. This promotes the decision commitment mentioned in Section 4, of which – in order to show effect – the bidders must be sure before submitting their offers. Concerning the final round, which comprises only the two winners of the first round, the bidders must be informed that only the last two will know each other's prices from the first round.

→ Recommended auction design: *FPSB – FPSB*

Constellation $n = 2$ or 3, low risk aversion, high σ:

If, as the only deviation from the previous case, a wide dispersion of the indifference prices is to be expected, the ex-post transparency in the first round should be restricted to telling the two winners which one of them bid lower ("rank information"), to prevent the winner from feeling too sure of himself and the loser from being discouraged. We refer to this version of a pay-as-bid auction with two winners as FPSB ^{R}, with 'R' for "'rank".

→ Recommended auction design: *FPSB* ^{R} *– FPSB*

Constellation $n = 2$ *or* 3*, high risk aversion, low* σ*:*

If the bidders are strongly risk-averse, small strategic margins are to be expected, so the final round should in all cases be a first-price auction. As a Dutch auction, it can serve to further reduce those margins. Regarding the first round, if there are not enough bidders for a Hongkong auction, a FPSB auction is the best choice.

→ Recommended auction design: *FPSB – Dutch*

Constellation $n = 2$ *or* 3*, high risk aversion, high* σ*:*

With few bidders and high risk aversion, the dispersion of the indifference prices plays no role for the decision as to whether the final round should be a first-price or a second-price auction. Even if, with low M and low σ, the inequality $M < g(n) \cdot \sigma$ is questionable, both sides of it are small, and therefore so is any disadvantage from a first-price auction.

Informing the two bidders in the final round of each other's bids does not hurt the auction result even if those prices are expected to be widely dispersed. Due to his high risk aversion, a large price gap should additionally motivate the trailing bidder to improve his offer. Although the leading bidder learns about his large lead, he also knows that his competitor now knows his price and might make a move accordingly, so the leader is induced to move again himself.

→ Recommended auction design: *FPSB – Dutch*

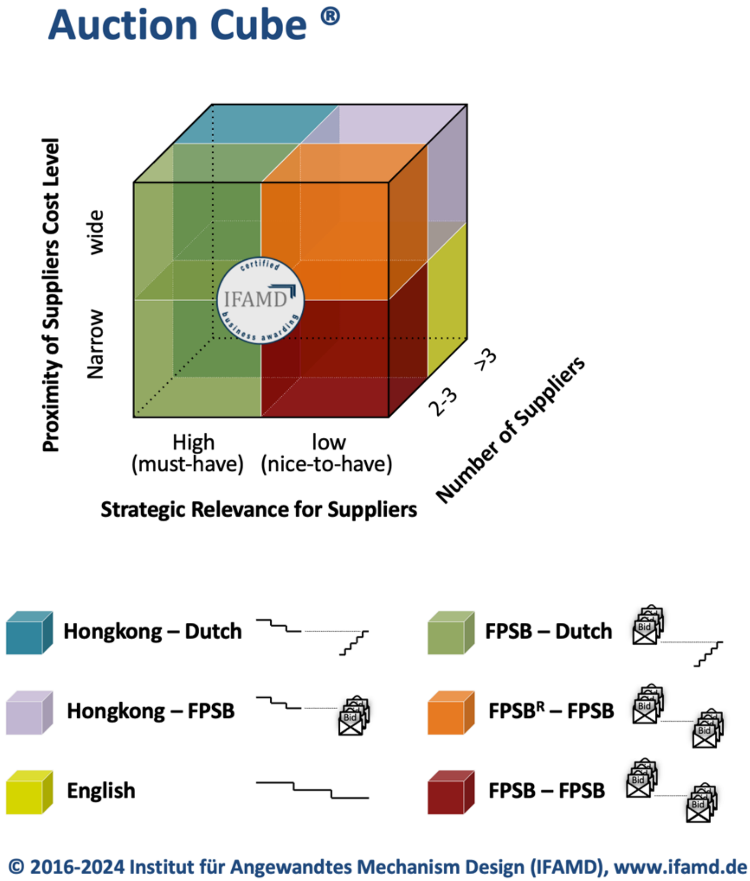


Figure 2: Overview of the auction designs recommended by the Auction Cube

6. Conclusion and prospects

The Auction Cube provides an unequivocal recommendation regarding the most suitable (compound) auction design for single-lot auctions with homogeneous and competitive bidders. If at least one of the three criteria that speak against holding an auction applies, other decision and negotiation mechanisms are to be preferred, such as take-it-or-leave-it-chains, take-it-or-auction offers, first-call options, last-call options, or offer rounds with sealed target prices that directly lead to a deal, to mention just the most important basic modes of negotiation that may become relevant in such circumstances. This is the case when the buyer has such strong internal preference for individual bidders that we can no longer speak of homogeneous bidders, or if bidder (implicit) collusion must be tactically addressed by the negotiation strategy. Finally, if more than one lot is up for negotiation, the whole field of combinatorial tenders opens up. Here the Auction Cube can provide orientation regarding the decision procedures within the individual lots. All that is left then for an additional meta-design is to define the synchronisation of the decision regarding the individual lots and admitted bundles of lots.

7. Notation

n	number of bidders in an auction
m	number of winners in an auction
σ	standard deviation of the bidders' indifference prices
$N(\mu, \sigma)$	normal distribution with expected value μ and standard deviation σ
$E(.)$	expected value of a probability distribution / of the result of an auction
X	order statistic of a normal distribution
$X(i, n)$	i^{th} order statistic of a normal distribution with sample size n
M	strategic margin in a first-price auction
N	strategic margin in a Hongkong auction
G	threshold of M in the decision between a first-price and a second-price auction, given a normal distribution
G'	threshold of M in the decision between a first-price and a second-price auction, given a uniform distribution and any σ
$g(n)$	factors for the calculation of G, depending on n and σ
Y	order statistics of the uniform distribution over $(0,1)$
Y''	order statistics of the uniform distribution over $(-1/2, 1/2)$
Y'	order statistics of the uniform distribution over $(-\sqrt{3} \cdot \sigma, \sqrt{3} \cdot \sigma)$
Z	order statistics of a normal distribution of indifference prices that are adjusted in response to competition
H	threshold of N in the decision between Hongkong auction and English auction, assuming a normal distribution
H'	threshold of N in the decision between Hongkong auction and English auction, assuming a uniform distribution and any σ
$h(n)$	factors for the calculation of H, depending on n and σ

Appendix

To calculate the expected values of the i^{th} order statistic with sample size n of the normal distribution $N(0,\sigma)$ with the density function f and the cumulative distribution function F, we use the formula for the density function $f_{X(i,n)}$ from David (2003: 10f):

$$f_{X(i,n)}(t) = \frac{n!}{(i-1)!\,(n-i)!}\, F^{i-1}(t)\,(1-F(t))^{n-i}\, f(t)$$

Using the integral formula for the expected values

$$E(X(i,n)) = \int_{-\infty}^{\infty} t \cdot f_{X(i,n)}(t)\, dt$$

we calculated the following values $g(n) = G/\sigma = E(X(2,n)) - E(X(1,n))$ and $h(n) = H/\sigma = E(X(3,n)) - E(X(2,n))$ in Maple, in each case for $\sigma = 1$.

n	$g(n)$	$h(n)$
2	1,128379167	n.a.
3	0,8462843752	0,8462843752
4	0,7323639907	0,5940227646
5	0,6679455035	0,4950189705
6	0,6254513222	0,440208205
7	0,5948041054	0,4046673114
8	0,5713754435	0,3794023676
9	0,5527157053	0,3603266738
10	0,537395686	0,3452979396
11	0,524519832	0,3330771153
12	0,513495456	0,3228939849
13	0,503912983	0,3142425616
14	0,495479279	0,3067755711
15	0,487978363	0,3002460517
16	0,48124717	0,294473127
17	0,475159993	0,289320999
18	0,469618166	0,28468553
19	0,46454302	0,280485393
20	0,459870964	0,276656044

21	0,455549958	0,273145545
22	0,451536949	0,269911572
23	0,447795969	0,266919251
24	0,444296702	0,264139556
25	0,441013388	0,261548117
26	0,43792398	0,259124294
27	0,435009481	0,256850484
28	0,432253419	0,254711569
29	0,429641436	0,252694478
30	0,427160944	0,250787853
31	0,424800858	0,248981765
32	0,422551369	0,247267496
33	0,420403765	0,245637348
34	0,418350273	0,244084505
35	0,416383935	0,242602898
36	0,414498498	0,24118711
37	0,412688328	0,239832285
38	0,410948335	0,238534054
39	0,409273898	0,23728848
40	0,407660818	0,236091997
41	0,40610527	0,234941372
42	0,404603757	0,233833662
43	0,403153074	0,232766188
44	0,401750286	0,231736494
45	0,400392687	0,230742334
46	0,399077784	0,229781648
47	0,397803278	0,228852536
48	0,396567039	0,227953248
49	0,395367095	0,22708217
50	0,394201615	0,226237806
51	0,393068897	0,22541877
52	0,391967354	0,224623778
53	0,39089551	0,223851632
54	0,389851984	0,22310122
55	0,388835483	0,222371503
56	0,387844801	0,221661512
57	0,386878802	0,220970341
58	0,385936425	0,22029714
59	0,385016671	0,219641113
60	0,384118596	0,219001515
61	0,383241319	0,218377643
62	0,382384001	0,21776884

63	0,381545856	0,217174481
64	0,380726141	0,216593981
65	0,379924149	0,21602679
66	0,379139217	0,215472383
67	0,378370712	0,214930268
68	0,377618037	0,214399977
69	0,376880623	0,21388107
70	0,376157933	0,213373127
71	0,375449453	0,212875749
72	0,374754698	0,21238856
73	0,374073202	0,211911201
74	0,373404524	0,211443332
75	0,372748244	0,210984628
76	0,372103961	0,210534779
77	0,371471289	0,210093492
78	0,370849864	0,209660486
79	0,370239337	0,209235493
80	0,369639372	0,208818258
81	0,369049651	0,208408536
82	0,368469867	0,208006094
83	0,367899726	0,207610708
84	0,367338948	0,207222165
85	0,366787263	0,20684026
86	0,366244412	0,206464797
87	0,365710147	0,20609559
88	0,36518423	0,205732455
89	0,364666432	0,205375222
90	0,364156533	0,205023724
91	0,363654321	0,204677802
92	0,363159593	0,204337301
93	0,362672152	0,204002077
94	0,36219181	0,203671985
95	0,361718385	0,20334689
96	0,361251701	0,203026661
97	0,36079159	0,202711172
98	0,360337889	0,202400301
99	0,359890441	0,202093929
100	0,359449092	0,201791945

References

Berz, G. (2015), *Game Theory Bargaining and Auction Strategies,* London: Palgrave-Macmillan (Org.: *Spieltheoretische Verhandlungs- und Auktionsstrategien*, 2007/2014/2022, Stuttgart: Schäffer Poeschel).

David, H.A, and H.N. Nagaraja (2003), *Order Statistics* (3rd edition), Hoboken, NJ: Wiley & Sons

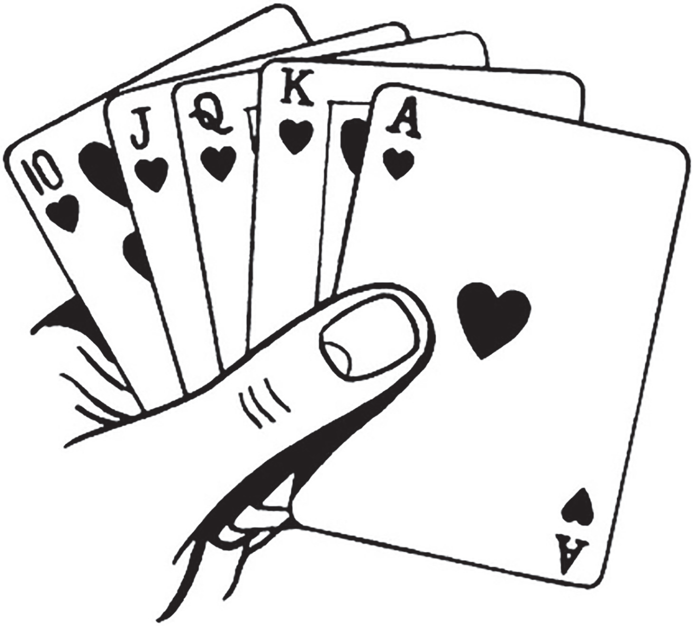
Holler, M. J., and B. Klose-Ullmann (2020), *Scissors and Rock: Game Theory for Those Who Manage,* Cham: Springer.

Klemperer, P. (2004), *Auctions: Theory and Practice,* Princeton: Princeton University Press.

Milgrom, P. (2004), *Putting Auction Theory to Work,* Cambridge, UK: Cambridge University Press.

Munich Social Science Review

New Series, Volume 8 / 2024



ACCEDO

Verlag Holler, München

Munich Social Science Review (MSSR)
New Series, Volume 8/2024

Applying Game Theory by Those Who Manage

Edited by
Gregor Berz & Florian Rupp

ACCEDO
Verlag Holler, München

IFAMD
game theory insights

Munich Social Science Review (MSSR), *New Series*
Volume 8/2024

Cover: Katharina Kohl
Cover image: CSA Images via Getty Images

Gnesener Str. 1, D-81929 München, Germany
www.ccr-munich.de, *manfred.holler@accedoverlag.de*

Print: Libri Plureos GmbH, Friedensallee 273, D-22763 Hamburg

Book orders can be placed with your local bookstore or with Amazon, etc.

ISSN 0170-2521
ISBN 978-3-88278-317-9

Munich Social Science Review (MSSR)
New Series, Volume 8/2024

Applying Game Theory by Those Who Manage

ISBN 978-3-88278-317-9
ISSN 0170-2521

9 783882 783179